Your Signature ____________________

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Please write your name on every page of this booklet and every additional sheet taken.

3. If you are using a Theorem/Result from class please state and verify the hypotheses of the same.

4. **Maximum time is 1.5 hours and Maximum Possible Score is 50.**

Score

Q.No.	**Alloted Score**	**Score**
1.	13	
2.	13	
3.	13	
4.	13	
Total	52	

Number of Extra sheets attached to the answer script: ____________________

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space.and $\{Y_n\}$ be a sequence of random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. Show that

$$E = \{\omega \in \Omega : \exists\, Y(\omega) \text{ such that } Y_n(\omega) \to Y(\omega) \text{ as } n \to \infty\}$$

is a measurable set.

2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space. Let $\{A_n\}_{n\geq 1}$ be a sequence of independent events. Suppose $X : \Omega \to \mathbb{R}$ is a random variable such that X is measurable with respect to the tail σ-field w.r.t $\{A_n\}_{n\geq 1}$. Show that there is a $\alpha \in \mathbb{R}$ such that $X = \alpha$ almost surely.

3. Let X_n be a sequence of independent random variables on $(\Omega, \mathcal{B}, P)$, such that $X_n \sim$ Exponential (a_n) with $a_n = \ln(n+1)$.

(a) (6 points) Show that the sequence converges to zero in probability.

(b) (7 points) Does the sequence converge to zero almost surely ?

4. Let $X \sim$ Normal (μ, σ^2), show that for $c > 0$

$$P(X - \mu \geq c) \leq \exp\left(-\frac{c^2}{2\sigma^2}\right).$$